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Balancing Medical Resident Education and Workload while Ensuring Quality Patient Care

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Balancing Medical Resident Education and Workload while Ensuring Quality Patient Care

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ABSTRACT

Medical residency is a requirement for medical professionals to practice medicine. Residency programs in internal medicine lasts 3 years and require residents to undergo a series of supervised rotations in elective, inpatient, and ambulatory units. Typically a team of chief residents develops a yearly rotational schedule that assigns residents to various departments for each week of the year, and for each day of the week. Scheduling resident rotations is complex as it needs to consider various academic, managerial, and legal restrictions while ensuring that the resulting schedules facilitate patient care and are balanced in terms of resident educational experience, workload, and resident satisfaction. This study proposes: (1) a multi-objective optimization approach for generating year-long resident rotation schedules; (2) an AHP (Analytical Hierarchy Process) model to compare schedules across multiple criteria and facilitate their adoption and implementation; (3) a methodology for studying the interaction between weekly and daily resident rotation schedules.; (4) an optimization based approach for ensuring continuity of care at outpatient clinics; and, (5) a methodology for evaluating resident assignment policies to outpatient clinics.

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1. INTRODUCTION

A typical medical residency program lasts 3 years, during which residents are required to practice medicine in a hospital system under the supervision of senior physicians. Overall, the residency is primarily an educational experience in patient-centered care that aims to enhance high quality resident education [1]. As part of the residency program, residents do rotations in various hospital units, and must also serve in a specific outpatient clinic, at which they return periodically after completing rotations in other units. Residents must also take two non-consecutive two-week vacations each year. In order to successfully manage the residency program, hospitals must first develop a year long schedule for its residents. Hospitals offering residency typically rely on a team of chief residents to develop this schedule.

Resident rotation schedules are typically developed without the help of decision support tools which lead to: (1) a time consuming process as it takes at least 2 months for chief residents to develop year long resident rotation schedules; (2) suboptimal schedules that do not necessarily balance resident education experience and work load distribution.

Inequitable workload distribution is a matter of concern in graduate medical residency as excessive working hours can lead to medical errors and affect patient quality care [2,3,12]. Volpp and Grande [3] conclude that residency schedules including on-duty periods of 36 hours or above impede adequate rest time between shifts, resulting in resident fatigue and increased potential of making medical errors. Volpp and Grande [3] also state that resident rotation schedules that reduce consecutive duty hours and distribute workloads more evenly results in fewer medication errors and better resource utilization [3]. Furthermore, Block et al [2] show that there is a clear relationship between resident rotation schedules and fatigue due to the incidence of factors like excessive night shifts and long working hours, which are associated with burnout and fatigue. Block et al [2] uses a cross sectional survey to show that hectic resident schedules (i.e., excessive workload and unbalanced work distribution) play a crucial role in cases of resident burnout and fatigue [2]. Similarly, West et al [16] claims that there is a relationship between resident fatigue and medical errors due to hectic working hours. Barger et al [20] surveys 2737 residents for medical errors committed by them and concludes that residents that have 5 or more night shifts in a month increase their odds of reporting at least 1 medical error due to fatigue by 7 times in comparison to those who did not work overnights. Therefore, there is a need to develop resident rotation schedules that are well balanced in terms of resident education, resident workload, resident satisfaction and ability to provide patient care under unplanned resident absence.

The need of balanced resident rotation schedules has been highlighted in the medical literature in the last few years. For example, Peets et al [4] highlights the need of a deeper understanding and analysis of the scheduling process so that the next generation of physicians can get the best compromise between education, experience, and patient care. For Sokie [17] one of the most important challenges for chief residents in rotational programs is to determine how to develop schedules that reduce fatigue and improve resident education while maintaining continuity of care. Fletcher et al [15] call for interventions that provided the right balance between continuity of care and physician fatigue to improve patient care.

Along these lines, in 2011, the (Accreditation Council for Graduate Medical Education) ACGME requested residency programs to design schedules that help minimize resident fatigue and augment patient safety [2]. In practice, residents' fatigue could be mitigated by reducing their number of working hours per week. However, this can affect continuity of care and patient safety in a negative way due to reduction in the resident's clinical exposure, unless residency program increase their enrollment.

The resident scheduling process typically consists of two stage. A first stage develops a weekly schedule and the second stage develops a daily schedule to satisfy the weekly requirements. A weekly schedule allocates residents to various units, outpatient clinics, and vacations for each of the 52 weeks in the year. Then, a more granular daily schedule allocates residents to shifts in specific units on a daily basis, while respecting their weekly assignments.

Resident scheduling is a complex process because of the internal policies, managerial practices and legal restrictions that must be simultaneously satisfied [1]. These restrictions are a combination of the ACGME guidelines and hospital's internal policies. Furthermore, there are resource limitations (number of senior physicians, equipment, time) that restrict the choice of solutions.

This study has been carried out in collaboration with the internal medicine resident rotational program at Rochester General Hospital (RGH) in Rochester, New York. The research conducted in this thesis also considers the additional needs and requirements specific to RGH.

Additionally the weekly and the daily schedules have their own set of restrictions. For example, planners must ensure that in the weekly schedule (1) each resident must have two two-week non-consecutive vacation periods per year; (2) residents must not undergo more than 3 consecutive weekly night shifts; (3) resident cannot be absent from their appointed outpatient clinics for more than 4 consecutive weeks; (4) certain units must host residents each week; (5) for each patient, a third of all the rotational time must be spent in Elective units, a third in Ambulatory units, and a third in Inpatient units; (6) each unit establishes the minimum and maximum numbers of year 1, 2, and 3 residents required for a week of rotation; (7) each unit also determines the minimum and maximum durations for each schedule rotation;

The daily rotational schedule must ensure that (1) residents do not work more than 6 nights in a row; (2) there must be at least 10 hours of "off time" between two consecutive shifts; (3) there should not be any 24 hour shifts for any resident; (4) resident duty hours must not exceed 80 hours averaged over a four week period; (5) if a resident is assigned to a night shift in a particular day, he/she cannot be assigned a shift starting the morning of the following day; (6) on average each resident must have one day off every 7 days; Schedulers must not only attempt to satisfy all the above restrictions, but also try to ensure that resident vacations are scheduled according to the resident's preferences.

Schedule planners need to ensure that throughout the residency year, each resident rotates in an outpatient clinic once every *n* number of weeks. Currently residents rotate for 4 weeks in other units and then do a rotation in their respective clinic for a week. This particular requirement ensures continuity of care at outpatient clinics since all residents come back to their respective clinics at regular intervals of time. To further ensure continuity of care at outpatient clinics at our partner hospital, year 1, 2 and 3 residents are grouped in 5 clinic groups which are teams of residents that coordinate patient care for a group of patients. It is needed that at least one member of each group must be present in the clinics, ensuring that if a patient is seen by a resident, his/her group members are aware of the case and can take care of the case, if needed, facilitating continuity of care. Clearly, incorporating all of the above complexity in a functional schedule by using a trial and error approach is unlikely to result in a better quality schedule, let alone a well balanced one.

This study, aims at answering the following research questions to understand how to facilitate the resident scheduling process and the generation of balanced schedules. In particular this study is interested in addressing the following questions: (a) how to determine a weekly rotation schedule that best balances resident education, workload, resident satisfaction and ability to provide patient care under unplanned resident absence? (b) how to develop daily resident rotation schedules? (c) how to better coordinate the integration between weekly and daily schedules? (d) how to ensure better continuity of care at outpatient clinics? (e) how to determine the best policy for assigning residents to outpatient clinics?

The main contributions of this study are: (1) a multi-objective optimization approach for generating year-long resident rotation schedules; (2) an AHP (Analytical Hierarchy Process) model to compare schedules across multiple criteria and facilitate their adoption and implementation; (3) a methodology for studying the interaction between weekly and daily resident rotation schedules.; (4) an optimization based approach for ensuring continuity of care at outpatient clinics; and, (5) a methodology for evaluating resident assignment policies to outpatient clinics.

The remaining of this document is structured as follows. Section 2 explores the literature on optimization-based approaches to resident scheduling. Section 3 presents the overall methodology overview for answering the aforementioned research questions. Section 4 presents a multi-objective optimization approach for developing year long weekly rotation schedules.

Section 5 provides details on developing daily rotation schedules. Section 6 discusses the approach for reducing infeasibility issues in weekly and daily rotation schedules. Section 7 presents an approach for ensuring better continuity of care at outpatient clinics. Section 8 discusses how to decide the best clinic policy for assigning residents to outpatient clinics. Finally, section 9 and 10 provide conclusions and future extensions respectively.

2. LITERATURE REVIEW

The nature of the resident scheduling problems has attracted the attention of the Operations Research community. However, current efforts in the area have failed to be generalizable to different hospitals, and have primarily focused on generating feasible or nearly feasible schedules for either the weekly assignments, or for the daily assignments. Moreover these approaches have focused mainly in satisfying the staffing needs of hospitals and have not paid sufficient attention on generating solutions that will result in more balanced schedules for the residents.

In this context, Franz and Miller [5] consider the weekly resident scheduling problem as a specific case of multi-period staff assignment problem that aims to maximize residents schedule preferences. Franz and Miller propose a linear programming approach that integrates feedback from the decision makers for determining if a generated schedule is acceptable or not. Once a feasible solution is achieved, a rounding heuristic approximates an LP solution to integer values. This heuristic provides a sub-optimal solution by rounding off the variable values for the most restrictive constraints.

A number of studies rely on mathematical programming models that integrate hard and soft constraints, where solutions must satisfy hard constraints and are allowed to deviate from satisfying soft constraints but are penalized for such deviations. For instance, Bard et al [11] proposes a schedule focusing only on assigning residents to outpatient clinics where a mixed integer programming approach maximizes clinic assignments. Topaloglu et al [6] incorporates a similar approach while proposing a goal programming model for scheduling EMRs (Emergency research Medical Residents) and utilizes the application of Analytical Hierarchy Process (AHP) for categorizing the constraints as soft and hard. Although Topaloglu et al [6] adopt a methodology that perfectly demonstrates the reduction in effort for developing high quality schedules, the assessment of a 'good schedule' does not incorporate the perceptions and preferences of the residents. In 2009, Topaloglu et al [19] proposed a model for developing the daily shift resident rotational schedule, while considering different resident seniority levels and applying a multi-objective mathematical programming to incorporate hard and soft constraints.

Day et al [18] use a mixed integer programming approach to develop a daily shift schedule for assigning medical residents to units over a 2 week period. The study [18] primarily focuses on the implementation of the 80-hour work week requirement and consider a system where residents rotate through 4 different hospitals. Topaloglu et al [8] also use a mixed integer programming model and a column generation approach to address the issue of daily shift resident rotation schedules. Again the assessment of the resulting schedules does not factor resident satisfaction as a criteria for assessing their quality.

In the context of integrating both weekly and shift schedules, Guo et al [7] provides a year long schedule generated through a three stage approach via a greedy algorithm, an integer programming model, and the exploration of alternative optimal solutions. This is the first study that aims not to overwhelm residents and meet duty hour standards. However, it is limited to a one year schedule and it does not incorporate rotations already performed by the residents. Cohn et al [9] uses a combination of mathematical programming and feedback heuristics for generating daily schedules. It allows chief resident interaction and feedback into the schedule development. Cohn et al [9] incorporates restriction and feedback from users and adapts its model to tailor solutions for individual chief residents (planners). However, Cohn et al [9] does not consider the need to incorporate restrictions regarding work life balance, as it only considers the hospital's staffing needs in generating the schedules.

Smalley and Keskinocak [10] propose an optimization-based decision support system that generates weekly as well as daily schedules. They also express the relation between quality of patient care and various factors like resident education, continuity of care, and resident duty hours but do not provide any methodological approach for the assessment and evaluation of schedules based on those factors. The authors propose a multi-objective problem that aims to minimize violations of not meeting the resident's preferences for being assigned to a given unit. In the system they also penalize the violations of unit demands, expressed by the number of residents by year required by the units. The last objective in the system aims to ensure experiential equity among the residents.

This proposed study, although closely related to Smalley and Keskinocak, differs from it in many aspects. For example, it uses a goal programming approach for satisfying multiple objectives that represent various aspects of a balanced schedule, presents a system for the assessment of how balanced schedules are, and it tries to balance the overall resident rotation schedule across all the residents over factors that are not just restricted to education experience.

3. METHODOLOGICAL OVERVIEW

This section presents an overview of the proposed methodological approach developed for answering the research questions discussed in Section 1.

We propose a two-stage approach where we first determine the weekly rotational schedule, and use the resulting plan as an input for determining the actual daily schedule for all residents. A mixed integer optimization problem is proposed for each of these stages. A balanced weekly resident rotation schedule is developed from a multi-objective optimization problem using goal programming. A detailed explanation of the approach, mathematical model, experimentation and results is given in Section 4. In addition to the aforementioned approach, the weekly schedule is in itself solved twice. First it is used to determine when clinic rotations, and ICU rotations must take place. Then we resolve the problem by fixing these key assignments and solving for the schedule for the remaining rotations. This way we are able to develop weekly schedules much faster with respect to solve times.

The resulting weekly schedule is then used as an input to a second optimization model that determines in which shifts of the day, residents must satisfy their weekly rotational assignments. Section 5 describes this in detail with the proposed mathematical model.

With weekly and daily schedules developed, we study the interaction between weekly and daily schedules, and develop a methodology for reducing infeasibility issues resulting from such integration. A more detailed explanation with the experimentation and results is available in Section 6.

The continuity of care at outpatient clinics is thereafter studied and a binary integer program is developed for optimizing continuity of care at outpatient clinics. The results show the optimal number of clinic groups and resident assignments to these groups. The model and results are available in detail in Section 7.

Finally, we propose a methodological approach for comparing outpatient clinic policies using a special case of the weekly schedule mathematical model, simulation and AHP(Analytical Hierarchy Process). The proposed approach, experimentation, and results can be seen in Section 8.

Figure 3.1 illustrates the above mentioned methodological overview. A detailed explanation of the notation and mathematical models used for the various research problems is available in their respective sections as mentioned above.

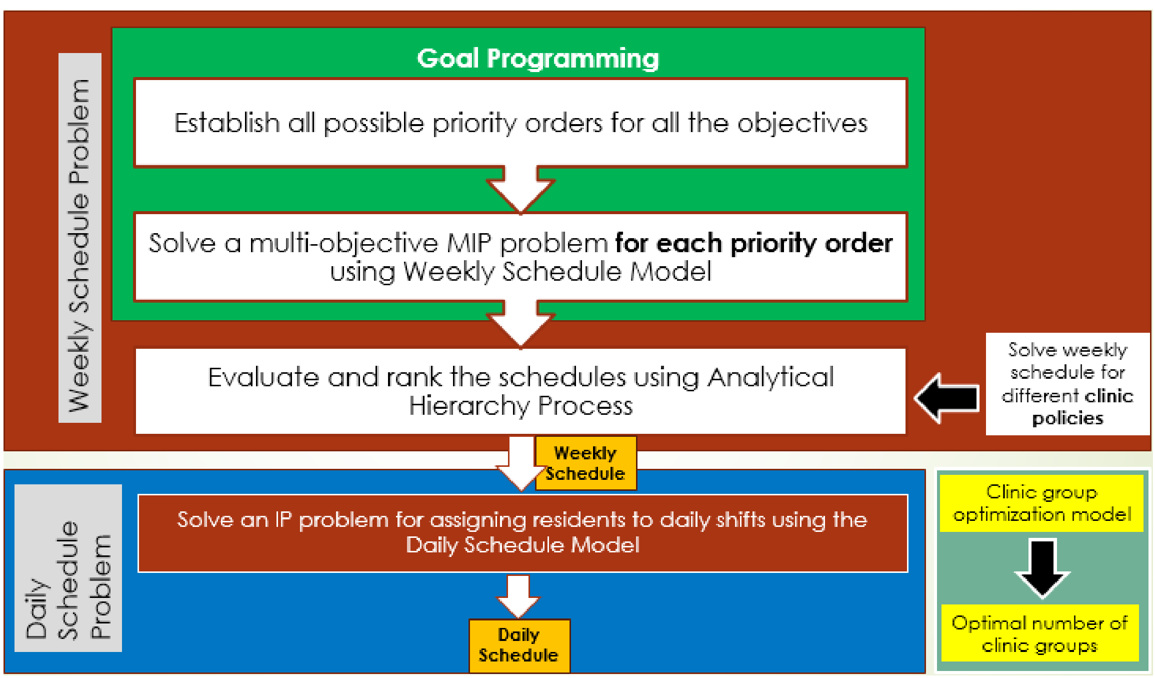


Figure 3.1 Weekly and Daily schedule generation methodology

4. A MULTI-OBJECTIVE OPTIMIZATION APPROACH FOR DEVELOPING BALANCED WEEKLY ROTATION SCHEDULES

4.1 Methodology

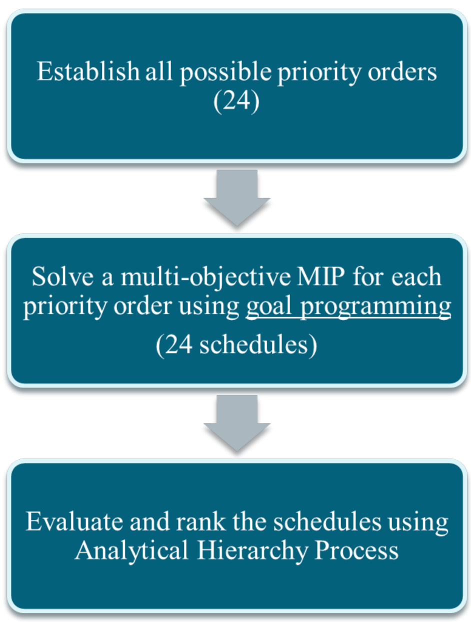
Resident rotation schedules should not only comply with the various legal, managerial and academic guidelines but should also balance resident workload, resident education experience, resident satisfaction and ability to provide patient care under unplanned residence absence. Therefore, we propose a multi-objective optimization problem that aims to balance these concerns. The following table describes a relationship between the concerns and their corresponding objectives used to quantify the concerns:

Table 4.1 Concerns and Objectives

Concerns	Objectives
Workload	Minimize night shift rotations
Resident education experience	Maximize elective rotations
Resident satisfaction	Maximize matching resident vacation preferences
Ability to provide patient care under unplanned resident absence	Maximize sick call Rotations

Given the objectives in table 4.1, we adopt a goal programming methodology for solving this multi-objective optimization problem. Goal programming problem requires deciding the priority order of the objectives under consideration. The priority order of objectives is the order in which goal programming will be implemented for the 4 objectives. Typically, this priority order is pre-established and then the problem is solved accordingly. In our case, there is no preferred priority order since we do not wish to bias towards any particular objectives, instead we solve the problem for all possible priority orders (24 in this case). Once the orders are established, a multi-objective mixed integer program is solved for each order, each resulting in a different weekly schedule, hence resulting in a total of 24 different balanced schedules. Finally Analytical Hierarchy Process (AHP) is used for evaluating and ranking these 24 schedules. Figure 4.1 shows the overall methodology for finding the most balanced schedule amongst the various possible schedules that can be developed using different priority orders.

This study uses Analytical Hierarchy Process for comparing and ranking multiple resident schedules based on the evaluation criteria described in Table 4.2. AHP is a useful tool for making decisions when there are multiple alternatives to choose from and there are various criteria on which these alternatives can be evaluated on. Hence this study uses AHP for selecting the best balanced schedule. The methodology is applied to compare and rank groups of schedules designed under specific experimental set up configurations.



The general AHP architecture used in analyzing M alternatives schedules over N criteria is illustrated in the figure below (Note: 'Alt.' = Alternative):

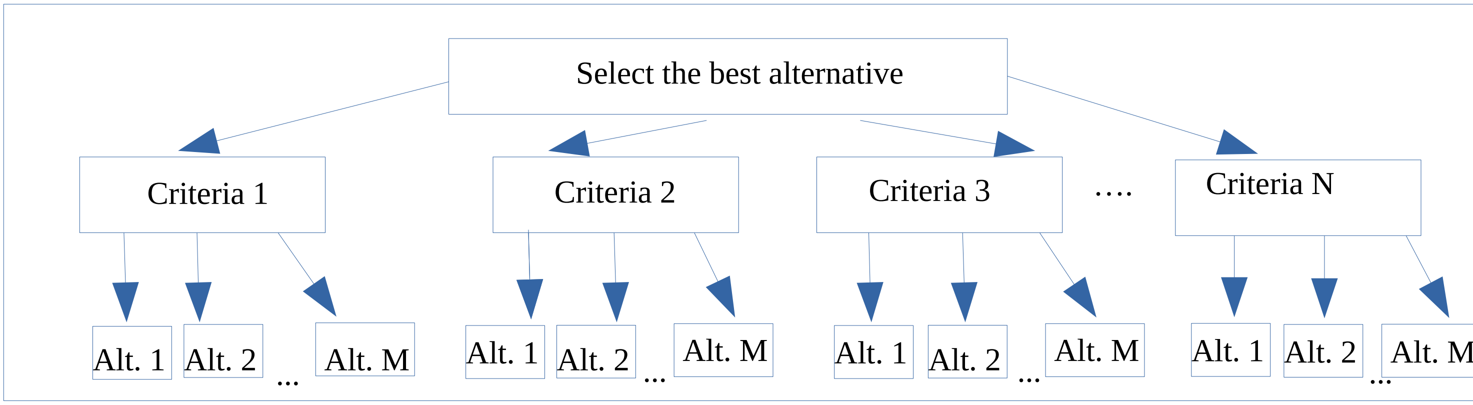


Figure 4.2 AHP generic architecture

The topmost level of the above figure depicts the overall target of the AHP, which in our case is selection of the best schedule. The second level represents the criteria on which the alternative schedules will be evaluated on. These criteria is described in Table 4.2. Finally, the alternatives in third layer of Figure 4.2 are the schedules under comparison. A detailed explanation of the AHP is provided in the Appendix II.

To apply the AHP for the 24 schedules generated previously, we collect 4 criteria statistics from each of them that describe how key objectives were implemented in the rotations of each resident during the academic year. The criteria selected for the AHP are described in the table below:

Table 4.2 Criteria for AHP

Criteria	**Justification for selecting the criteria**
Variance in the number of night shift rotations across all residents in a year	Low variance = better workload distribution
Variance in the number of elective rotations across all residents in a year	Low variance = better education experience

Variance in the number of sick call rotations across all residents in a year	Low variance = better ability to provide patient care under unplanned resident absence
Overall vacation preference satisfaction for all residents in a year	Higher the value = higher resident satisfaction

The aforementioned criteria were selected because these criteria can be used to characterize balanced schedules. In particular, sick-call rotations refer to rotations where residents are put on-call to replace other residents, in case anyone misses its shift. It must be noted here that the residents who are on sick-call do not sit idle, instead, they are assigned elective rotation where they continue to work unless they are called to fill the place of an absent resident. Given that night shifts are one of the primary causes of resident fatigue (as shown in section 1), and potentially a cause of medical errors, the resident workload distribution can be measured as a proxy by the distribution of weeks in night shifts among all residents over a year. Schedules that result in high variability on the distribution of night shift rotations have poorer quality than schedules with less variable distributions for this rotation.

We also map the quality of the rotational program by the ability that residents have to experience a wide variety of unit rotations. This flexibility and level of exposure is directly related to the number of elective rotations a resident can have during the program. Elective rotations is collective term for a bunch of units from which the residents can choose to rotate. In other words, whenever a resident is assigned an elective, she/he may choose a unit from a number of units, where he/she might wish to rotate. Clearly, higher the elective rotations for a resident, higher is their chance of doing rotations in a wide variety of units, thereby defining their education experience. Thus for a balanced education experience, the number of electives shouldn't be high, rather they should be equitable amongst all residents. The lower the variance in the number of elective rotations in a schedule among all residents in year, the better the schedule. A proxy to the ability to provide patient care under unplanned residence absence is given by the number of sick-call rotations. Therefore, lesser the variance in the number of sick-call rotations in a schedule over a year, the less likely that the hospital will have resident shortages in any given week. Finally, the residents' overall vacation preference satisfaction measures how well a schedule tries to accommodate the residents vacation preferences.

In the following sub section we provide a detailed mathematical model of the MIP formulation used to generate a schedule for a specific priority order. The constraints are implemented as hard and soft

4.2 Weekly Schedule Model (WSM)

Let U be the set of units for which the weekly schedule needs to be solved. The WSM is solved in two steps. Initially the model is solved for a set of units Θ, which includes clinic units (i.e. TWIG and OPD) and the intensive care units (ICUs) (i.e., MICU_D and MICU_N). The resulting schedule is then fixed and the optimization model is resolved considering all remaining rotational units. A description of all the rotational units is available in Appendix I.

Sets

Y academic residency years (1, 2, 3)
R all residents
R_i residents in academic year i, $i \in Y$, $R_i \subset R$
C clinic units (TWIG, OPD)
G clinic groups
T weeks in academic program during a year, $T = \{1, \ldots, np\}$
V vacation units (a single-unit set)
Q units that require only 1 rotation per week
A ambulatory units
I inpatient units
E elective units
N units that require residents in rotation every week
γ artificial variables
H_u type of resident allowed to rotate in unit $u \in U$
U all units being considered for solving weekly schedule
Θ key units : {TWIG,OPD,MICU_D,MICU_N}
UN night shift units
SC sick call
P clinic policies

Parameters

Π_p number of weeks within which a resident must return to a rotation in his/her clinic for policy $p \in P$
np the total number of weeks in the academic year (52)
$h_{g,c}$ binary; 1 if resident group $g \in G$ is assigned to clinic $c \in C$, and 0 o.w.
$l_{r,g}$ binary; 1 if resident $r \in R$ is part of group $g \in G$, and 0 o.w.
ζ_u number of weeks of rotation required over 3 years in unit $u \in U$, where, $\zeta^{min} \leq \zeta_u \leq \zeta^{max}$
α_u number of weeks of rotation in unit $u \in U$ required in a year, where, $\alpha_u^{min} \leq \alpha_u \leq \alpha_u^{max}$
λ_u number of weeks in continuous rotation in unit $u \in U$ where, $\lambda_u^{min} \leq \lambda_u \leq \lambda_u^{max}$
$\Psi_{r,t}$ preference of resident r for having vacation at period $t \in T$
$\Phi_{i,u}$ number of residents in a year $i \in Y$ required in unit $u \in U$ per week, where $\Phi_{i,u}^{min} \leq \Phi_{i,u} \leq \Phi_{i,u}^{max}$
τ minimum number of clinic rotations for each resident
$\omega_{r,u}$ rotations completed by resident $r \in R$ in unit $u \in U$ in previous academic years
v minimum gap in weeks between 2 vacation blocks
s number of clinic rotations in the first Π_p weeks
m minimum number of residents in a clinic group each week

Decision Variables

$X_{r,u,t}$ binary; 1 if resident $r \in R$ is assigned to unit $u \in U$ in week $t \in T$, 0 o.w.
$W_{r,u,t}$ binary; 1 if resident $r \in R$ starts a rotation in unit $u \in U$ in week $t \in T$, 0 o.w.
$\Delta_{u,t}$ binary; 1 if unit u starts a rotation in period t, 0 o.w.

Objective Function

$$\text{Maximize} \sum_{r \in R} \sum_{u \in V} \sum_{t \in T} \Psi_{r,t} X_{r,u,t} \tag{0}$$

$$\text{Minimize} \sum_{r \in R} \sum_{u \in UN} \sum_{t \in T} X_{r,u,t} \tag{1}$$

$$\text{Maximize} \sum_{r \in R} \sum_{u \in E} \sum_{t \in T} X_{r,u,t} \tag{2}$$

$$\text{Maximize} \sum_{r \in R} \sum_{u \in SC} \sum_{t \in T} X_{r,u,t} \tag{3}$$

The multiple objectives for the multi-objective optimization problem are maximizing matching vacation preferences, minimizing night shift rotations, maximizing elective rotations and maximizing sick call rotations while minimizing the sum of penalties on all soft constraints.

Constraints

Hard Constraints

Constraints 4,5, and 6 ensure that assignments satisfy any fixed or pre-assigned rotations.

$$W_{r,u,t} \geq W^o_{r,u,t} \qquad \forall \ r \in R, \ u \in U, \ t \in T \tag{4}$$

$$X_{r,u,t} \geq X^o_{r,u,t} \qquad \forall \ r \in R, \ u \in U, \ t \in T \tag{5}$$

$$\Delta_{u,t} \geq \Delta^o_{u,t} \qquad \forall \ u \in U, \ t \in T \tag{6}$$

Constraint 7 ensures that for each resident, clinic rotations happen periodically and that a minimum number of clinic rotations happens in the academic year.

$$\sum_{n=1}^{\alpha_c^{min}} X_{(r,c,t+\Pi_p n)} \geq W_{r,c,t} \qquad \forall\ r\in R,\ t\in\{1..\Pi_p\},\ c\in C \tag{7}$$

Constraint 8 ensures that any given interval of Π_p consecutive weeks during the academic year, exactly s clinic rotation takes place for every resident.

$$\sum_{t}^{t+\Pi_p-\lambda_c} X_{r,c,t} = s \qquad \forall\ r\in R,\ t\in\{1..np-\Pi_p+\lambda_c\},\ c\in C,\ g\in G,\ p\in P \tag{8}$$

Constraint 9 ensures that during the first Π weeks of the academic year, each resident has to start exactly s clinic rotation.

$$\sum_{c\in C}\sum_{t=1}^{\Pi_p} W_{r,c,t} = s \qquad \forall\ r\in R \tag{9}$$

Clinic groups are pre-assigned to the clinic units, so that residents in a group only do clinic rotations in one particular clinic for the three years of the academic program. Constraint 10 ensures that each resident is allocated to one type of clinic throughout the year.

$$\sum_{g\in G} l_{r,g} h_{g,c} - X_{r,c,t} \geq 0 \qquad \forall\ r\in R,\ t\in T,\ c\in C \tag{10}$$

Constraint 11 ensures that there are at least m residents from each clinic group starting a clinic rotation in each of the first Π_p weeks.

$$\sum_{r\in R}\sum_{c\in C} l_{r,g} W_{r,c,t} \geq m \qquad \forall\ t\in\{1..\Pi_p\},\ g\in G \tag{11}$$

Constraint 12 ensures that there is at least one resident in a clinic rotation each week from each group.

$$\sum_{r\in R} X_{r,c,t}\, l_{r,g} \geq 1 \qquad \forall\ g\in G,\ c\in C,\ t\in T \tag{12}$$

Constraint 10 ensures that in every period, each unit receives up to the maximum number of year

i (1,2 or 3) residents.

$$\sum_{r \in R_i} X_{r,u,t} \leq \Phi_{i,u}^{max} \quad \forall \ u \in U, \ t \in T, \ i \in Y \tag{13}$$

Constraint 11 ensures that each resident is assigned to only one unit per week.

$$\sum_{u \in U} X_{r,u,t} = 1 \quad \forall \ r \in R, \ t \in T \tag{14}$$

Constraint 12 ensure that residents do not do rotations in units where they are not allowed to rotate.

$$\sum_{r \in R_j} X_{r,u,t} = 0 \quad \forall \ u \in U, \ j \notin H_u, \ t \in T \tag{15}$$

Constraint 13 ensures that residents are allocated to units only if they start a rotation in the unit in that week or a prior week within the expected duration of a rotation in the unit.

$$X_{r,u,t} - \sum_{tt \in (t-\lambda_u^{min}+1)..t} W_{r,u,tt} = 0 \quad \forall \ u \in U, \ j \in H_u, \ r \in R \quad t \in \{\lambda_u^{min}..np\} \tag{16}$$

Constraint 14 ensures that no more than 3 consecutive night shifts weeks are allocated to any resident throughout the 52 week period.

$$\sum_{u \in UN} \sum_{t_o \in t..t+3} X_{r,u,t_o} \leq 3 \quad \forall \ r \in R, \ t \in \{1..np-3\} \tag{17}$$

Soft Constraints

Constraint 18 to 20 ensure that every year during the number of working weeks (i.e., excluding vacations) one third of all rotations for each resident happen in Ambulatory units, one third in Elective units and one third in Inpatient units. The implementation of these constraints serves as a proxy for ensuring that a third of all rotations in a three year program are spent in Ambulatory, Elective, and Impatient units, respectively.

$$\sum_{t\in T}\sum_{u\in A} X_{r,u,t}+\sum_{u\in A}\omega_{r,u} \;\geq\; (3np-12)/3 \qquad \forall\; r\in R \tag{18}$$

$$\sum_{t\in T}\sum_{u\in E} X_{r,u,t}+\sum_{u\in E}\omega_{r,u} \;\geq\; (3np-12)/3 \qquad \forall\; r\in R \tag{19}$$

$$\sum_{t\in T}\sum_{u\in I} X_{r,u,t}+\sum_{u\in I}\omega_{r,u} \;\geq\; (3np-12)/3 \qquad \forall\; r\in R \tag{20}$$

Constraint 21 ensures that at least 1 rotation happens each week for units in set N

$$\sum_{r\in R_j} X_{r,u,t} \;\geq\; 1 \qquad \forall\; u\in N,\; t\in T,\; j\in \mathrm{H}_u \tag{21}$$

Constraints 22 ensure that during each year, each resident completes required minimum and maximum rotations in each unit respectively.

$$\alpha_u^{min} \;\leq\; \sum_{t\in 1..np} X_{r,u,t} \;\leq\; \alpha_u^{max} \qquad \forall\; u\in U,\; j\in H_u,\; r\in R_j \tag{22}$$

Constraint 23 ensures that in every period, each unit receives at least a minimum number of year 1,2 and 3 residents.

$$\sum_{j\in \mathrm{H}_u}\sum_{r\in R_i} X_{r,u,t} \;\geq\; \Phi_{i,u}^{min} \qquad \forall\; u\in Q,\; t\in T,\; i\in H_u \tag{23}$$

Constraint 24 is used to ensure that any prior rotation done by residents in years 2, and 3 is taken into account towards this year's rotational schedule. We ensure that we do not over assign residents to unit when they have already completed the maximum number of permissible rotations in the three year program.

$$\zeta_u^{min} \;\leq\; \sum_{t\in T} X_{r,u,t}+\omega_{r,u} \;\leq\; \zeta_u^{max} \qquad \forall\; u\in U,\; j\in H_u,\; r\in R_j \tag{24}$$

Constraint 25 ensures that if a rotation takes place for any resident, then the rotation lasts the minimum continuous rotational time.

$$\sum_{(tt \in t..t+\lambda_u^{min}-1 \mid t+\lambda_u^{min}-1 \leq np)} X_{r,u,tt} \geq W_{r,u,t} \lambda_u^{min} \qquad \forall \; u \in U, \; j \in H_u, \; r \in R_j \quad t \in T \mid \lambda_u^{min} > 1 \tag{25}$$

Constraint 26 ensures that the two non-consecutive two-week vacation periods are not consecutive.

$$W_{r,u,t} + W_{r,u,t+2} \leq v \qquad \forall \; r \in R, \; u \in V, \; t \in 1..np-2 \tag{26}$$

Minimizing sum of all penalties is not shown in the above objective function for simplification purposes. The full WSM with the overall objective function can be seen in Appendix IV.

The WSM becomes infeasible if all the constraints are treated as hard constraints. Thus the constraints are carefully divided into hard and soft constraints by analyzing the feedback from the chief residents at RGH on the importance of each individual guideline. Guidelines that cannot be deviated at all are taken as hard constraints while guidelines where relaxations are possible are taken as soft constraints. Among the soft constraints, relative importance is decided based on a numerical feedback from the chief residents again. A 1-10 point scale is used where 1 implies that maximum relaxation is allowable while 10 implies minimal relaxation is allowable. For the soft constraints in this model(Constraints 15-23) we add an artificial variable for each of them. This is done to account for any infeasibility occurring in the system due to these constraints. Penalties are enforced on each of these artificial variables and the sum of all these penalties is minimized in the objective function. For simplification purposes, we have omitted showing the artificial variables and the penalties here, but all the constraints with the artificial variables and penalties can be seen in Appendix IV.

4.3 Experimentation and Results

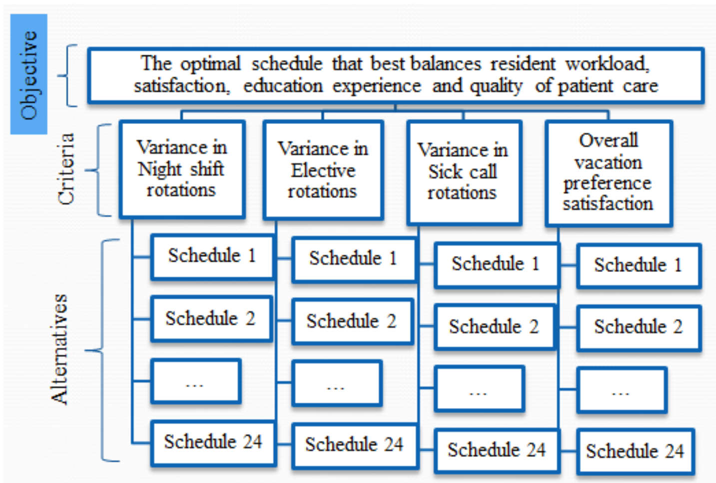
We utilize goal programming for solving this multi-objective optimization problem. Given our 4 objectives: Maximizing matching resident vacation preferences(V); Minimize night shift rotations(N); Maximizing elective rotations(E); Maximizing sick call rotations (S); we can establish 24 different priority orders for these 4 objectives. For each established priority order we solve the multi-objective MIP model as a goal programming problem and collect data on the criteria described in section 4.1.

Table 4.3 shows the data collected from running the MIP model described in section 4.2 for all 24 possible priority orders using the RGH data appended in appendix V.

Table 4.3 Data from analyzing the 24 schedules developed by goal programming

Order No.	Priority Order	Overall vacation preference satisfaction	Variance in sick call rotations	Variance in night shift rotation	Variance in elective rotation
1	V-E-N-S	1219.0	1.5	2.7	11.4
2	V-N-E-S	1210.0	1.4	2.1	12.7
3	V-E-S-N	1158.0	1.1	3.9	10.9
4	V-N-S-E	1197.0	1.2	1.8	13.5
5	V-S-E-N	1200.0	0.8	3.5	12.3
6	V-S-N-E	1205.0	0.7	2.4	13.7
7	S-V-E-N	1012.0	0.3	4.5	12.8
8	S-V-N-E	1045.0	0.4	3.9	13.9
9	S-E-V-N	910.0	0.3	5.1	11.8
10	S-E-N-V	840.0	0.5	3.6	12.4
11	S-N-E-V	896.0	0.6	2.8	13.2
12	S-N-V-E	923.0	0.5	2.6	14.3
13	N-V-E-S	992.0	1.9	0.6	14.6
14	N-V-S-E	973.0	1.5	0.5	15.3
15	N-S-E-V	795.0	1.3	0.6	14.1
16	N-S-V-E	877.0	1.3	0.8	15.8
17	N-E-S-V	805.0	1.6	0.4	13.2
18	N-E-V-S	884.0	1.8	0.6	13.4
19	E-V-N-S	1117.0	0.9	2.1	8.2
20	E-V-S-N	1161.0	0.7	2.8	8.8
21	E-S-N-V	891.0	0.6	1.8	9.1
22	E-S-V-N	958.0	0.6	2.5	8.1
23	E-N-S-V	870.0	0.7	1.3	8.5
24	E-N-V-S	969.0	0.8	1.1	9.2

Following we use AHP as described in section 4.1 to obtain the best balanced schedule amongst these 24 schedules. The Following figure shows the structure of AHP for this problem:



The AHP is then conducted with the data in table 4.3. It must be noted that all pairwise comparison matrices for each of the criteria are populated using the aforementioned data from the experimentation. The AHP consists of several comparison matrices, one of which is the criteria comparison matrix. As mentioned earlier in this section, since there is no bias for any particular priority order of objectives(criteria) for solving the multi-objective optimization problem, we treat their relative importance to each other equally and henceforth the following matrix is developed:

Legend: C1: Variance in the number of night shift rotations over all residents in a year

C2: Variance in the number of elective rotations over all residents in a year

C3: Variance in the number of sick call rotations over all residents in a year

C4: Overall vacation preference satisfaction for all residents in a year

Judgement Matrix for the 4 Criteria	**C_1**	**C_2**	**C_3**	**C_4**
C_1	1	1	1	1
C_2	1	1	1	1
C_3	1	1	1	1
C_4	1	1	1	1

$\lambda_{max} = 4$, C.I. = 0, C.R. = 0%

The C.R. value of 0% implies that this is a consistent matrix for the AHP.

The following order was obtained from the AHP:

Table 4.4 AHP results

Order Number	**Order Type**	**Normalized Weight**
17	N-E-S-V	100
14	N-V-S-E	83.6
15	N-S-E-V	82.4
9	S-E-V-N	81.2
7	S-V-E-N	78.6
18	N-E-V-S	78.3
23	E-N-S-V	77.8
24	E-N-V-S	75.0
13	N-V-E-S	74.6
21	E-S-N-V	74.0
22	E-S-V-N	71.9
16	N-S-V-E	69.5
8	S-V-N-E	67.3

10	S-E-N-V	67.2
12	S-N-V-E	65.7
19	E-V-N-S	63.9
20	E-V-S-N	62.9
11	S-N-E-V	62.6
6	V-S-N-E	55.6
5	V-S-E-N	52.2
4	V-N-S-E	51.7
3	V-E-S-N	49.9
2	V-N-E-S	49.2
1	V-E-N-S	47.9

The results show that amongst the 24 generated schedules the schedule that best balances our objectives follows the following priority order:

Minimize Night shift rotations (N) > Maximize elective rotations (E) > Maximize sick call rotations (S) > Maximize matching resident vacation preferences (V)

The model was solved using AMPL and Gurobi on an intel core i5 processor with 2 GB RAM. The solve time reported for solving a single multi-objective optimization problem was 2 hours.

5. DEVELOPING DAILY RESIDENT ROTATION SCHEDULES

5.1 Methodology

Daily schedules refer to the shift assignments of residents to various units. A binary integer program is developed (Daily Schedule Model) which develops daily assignments of residents to various units. The daily schedule can only be developed after the weekly schedule has been developed because the daily assignments of residents are based off the weekly assignments.

5.2 Daily Schedule Model (DSM)

This stage 2 model is responsible for allocating residents to the day and shifts in which they must satisfy their weekly rotational assignments. The weekly assignments is fed into this model as an input parameters. The following additional notation is used to describe the DSM.

Sets

DS set of units requiring day shifts
D set of days in a week
S set of types of shifts in all units
U^o set of units open seven days a week

Parameters

$h_{u,s}$ duration of shift s in unit u
$X_{r,u,w}$ binary parameter which is 1 if` resident r has been assigned to unit u in week w, 0 otherwise
$\varphi_{u,s}$ binary parameter which is 1 if a unit u has a shift s, 0 otherwise
$\alpha_{u,s}$ starting time of all shifts
$\beta_{u,s}$ ending time of all shifts
$\tau_{u,d}$ binary parameter which is 1 if a unit u is open on day d, 0 otherwise
Φ_u minimum number of residents required by unit u
π time parameter used for ensuring 10 hour gap between shifts
p maximum number of consecutive night shifts
e maximum amount working hours per week averaged over a four week period

Variables

$\xi_{r,u,w,d,s}$ 1 if a resident r is allocated to unit u in week w, day d, and shift s
0 otherwise

Objective Function

Maximize 0

The objective for the daily schedule is to obtain a feasible assignment of residents to daily shifts based on the weekly schedule developed from the Weekly Schedule Model. Hence, we keep the objective function a constant since all are guidelines are incorporated as the constraints.

Constraints

Constraint 27 ensures that the resident weekly assignments obtained from solving the WSM are satisfied while assigning residents to shifts.

$$\xi_{r,u,w,d,s} \leq X_{r,u,w}\tau_{u,d}\,\varphi_{u,s} \quad \forall \; r\in R, u\in U, w\in T, d\in D \tag{27}$$

Constraint 28 ensures that there is at least one shift assignment for every weekly assignment obtained from the weekly schedule.

$$\sum_{d\in D}\sum_{s\in S}\xi_{r,u,w,d,s} \geq X_{r,u,w} \quad \forall \; r\in R, u\in U, w\in T \tag{28}$$

Constraint 29 ensures that there are not more than p consecutive night shifts in a single week for any resident throughout the 52 week period.

$$\sum_{s\in S}\sum_{d\in D}\xi_{r,u,w,d,s} \leq p \qquad \forall \; r\in R, u\in N, w\in T \tag{29}$$

Constraint 30 ensures that there are not more than 6 consecutive night shifts in 7 consecutive days spread across two consecutive weeks for any resident throughout the 52 week period.

$$\sum_{s\in S}\sum_{u\in N}\sum_{d_o=d}^{min(7,d+6)} \xi_{r,u,w,d_o,s} + \sum_{s\in S}\sum_{u\in N}\sum_{d_o=1}^{d-1} \xi_{r,u,w+1,d_o,s} \leq p \qquad \forall \; r\in R,\, d\in\{2..7\},\, w\in T \tag{30}$$

Constraint 31 ensures that residents do not work more than 80 hours in any week.

$$\sum_{s\in S}\sum_{d\in D} h_{u,s}\xi_{r,u,w,d,s} \leq e \qquad \forall \; r\in R,\, w\in T,\, u\in U \tag{31}$$

Constraint 32 ensures that residents do not work more than 80 hours in 7 continuous days across any 2 consecutive weeks throughout the 52 week period.

$$\sum_{s\in S}\sum_{d_o=d}^{min(7,d+6)} h_{u,s}\xi_{r,u,w,d_o,s} + \sum_{s\in S}\sum_{do=1}^{d-1} h_{u,s}\xi_{r,u,w+1,d_o,s} \leq e \qquad \forall \; r\in R,\, d\in\{2..7\},\, w\in T,\, u\in U \tag{32}$$

Constraint 33 ensures that each resident gets a 10 hour gap between any two consecutive shifts in a day.

$$\sum_{u\in U,s\in S} \alpha_{u,s}\xi_{r,u,w,d,s} - \sum_{u\in U,s^o\in S} \beta_{u,s^o}\xi_{r,u,w,d,s^o} \geq \pi \qquad \forall \; r\in R, w\in T, d\in D \mid s\neq s^o \tag{33}$$

Constraint 34 ensures that there is a 10 hour gap between 2 consecutive shifts for each resident in a week.

$$\sum_{u\in U,s\in S} (\alpha_{u,s}\xi_{r,u,w,d+1,s} - \beta_{u,s}\xi_{r,u,w,d,s}) \geq \pi - 24 \qquad \forall \; r\in R, w\in T, d\in D \mid d+1\leq 6 \tag{34}$$

Constraint 35 ensures that for each resident there is a 10 hour gap between 2 consecutive shifts spread across 2 consecutive weeks.

$$\sum_{u \in U} \sum_{s \in S} (\alpha_{u,s} \xi_{r,u,w,d,s} - \beta_{u,s} \xi_{r,u,w+1,d-6,s}) \geq \pi - 24 \quad \forall \; r \in R, w \in T, d \in 7 \mid w+1 \leq 52 \tag{35}$$

Constraint 36 and 37 ensure that each resident gets a day off every 7 days while the minimum required residents in each unit at any given time are always present.

$$\sum_{r \in R, s \in S} \xi_{r,u,w,d,s} = \Phi_u - 1 \quad \forall \quad u \in U^o, w \in T, d \in D \tag{36}$$

$$\sum_{d \in D, s \in S} \xi_{r,u,w,d,s} = 6 \quad \forall \quad u \in U^o, w \in T, r \in R \tag{37}$$

6. REDUCING INFEASIBILITY ISSUES IN WEEKLY AND DAILY SCHEDULES

6.1 Methodology

The integration of weekly and daily schedules can result in infeasibe solutions. Thus, we propose a more efficient approach that combines analysis and optimization. The general methodology can be understood as a 3 step process:

Step 1: Identification of units that have the most restrictive requirements in both weekly and daily schedules. Identification of these units is done by checking each unit on four key traits: (1) If the unit must be in rotation each week; (2) If the unit has no flexibility in the minimum and maximum continuous rotation time; (3) If the unit has no flexibility in terms of minimum and maximum requirements for residents; (4) If the unit must be open 7 days of the week.

Step 2: The WSM(described in the Section 4.2) is slightly modified for the units identified for the previous step. The reformulation consists of correcting soft constraints for the identified units into hard constraints. As a result of this step, the reformulated weekly schedule gives priority satisfying the needs of the most restrictive units, hence reducing the likelihood of meeting overall infeasible solutions.

Step 3: Comparison of the schedule developed with this integrated approach and an uncoordinated approach. The comparison is made by evaluating the schedules on solve times, infeasibility value and effect on factors that represent balance in a schedule, that is, vacation preferences, night shift rotations, elective rotations, and sick call rotations.

Thus, overall the integrated approach of analyzing weekly and daily schedules as a single entity helps us to understand the key infeasibility areas in the weekly schedule that can cause corresponding infeasibilites in the daily schedule and as a result of this approach, we are able mitigate these potential infeasibilities by modifying the weekly schedule.

6.2 Experimentation and Results

The weekly and daily schedule models are analyzed for key restrictive units that can be potential sources of infeasibility. It is observed that the set 'N' (Set of units that require a rotation each week) which is described section 4.2, is a potential source of units that can be very restrictive in terms of requirements. When these units are analyzed on the 4 key traits mentioned in section 6.1, 5 units are found to be the most restrictive units. The following table shows the list of those 5 units:

Table 6.1 Most restrictive units

Units
MICU_D
MICU_N
MAT_D
MAT_N
Overnight

For the above units, the WSM (described in section 4.1) is altered as described in section 6.1. In particular, soft constraints 18,20 and 22 of the WSM are duplicated into hard constraints for the units in the table 6.1 Although it must be noted here that for simplification purposes we use the special case of our multi-objective model, i.e., we consider a single objective of maximization of vacation preferences while solving the model. The model is solved using AMPL and Gurobi on an intel core i5 processor with 2 GB RAM.

We compare the weekly rotation schedule developed from the integrated approach with an originally developed weekly rotation schedule on various factors and the following table shows the results obtained:

Table 6.2 Results of comparison of integrated approach schedule with original schedule

Comparison Metric	Schedule from Integrated Approach	Original Schedule	% increase/decrease from original schedule
Solve time	466.78 seconds	284.09 seconds	64.3 % increase
Infeasibility value	144488100	384570524	62.4 % decrease
Matching vacation preference satisfaction value	228	228	0 %
Variance in night shift rotations	4.7 weeks	9.1 weeks	48.3 % decrease
Variance in elective rotations	8.1 weeks	10.2 weeks	20.58 % decrease
Variance in sick call rotations	0.5 weeks	0.5 weeks	0 %

The above results show that there was an increase in solve times while a substantial decrease in the infeasibilities in the schedule generated from the integrated approach. Additionally, the variance in night shifts and elective rotations also reduce in the schedule generated from the integrated weekly and daily schedule approach. The variance in sick call and the matching vacation preference satisfaction value remained unchanged.

7. OPTIMIZATION OF CONTINUITY OF CARE AT OUTPATIENT CLINICS

7.1 Methodology

As described earlier in Section 1, all residents are pre-assigned to clinic groups. We adopt a more granular approach of developing clinic shift assignments for optimizing the number of clinic groups. Since on the weekly level, the clinic rotations repeat in a pattern (once every 5 weeks) for each resident, we develop a Binary Integer Programming (BIP) model and solve it for a week. Since clinics repeat at regular intervals of time, this single week of assignments can be used for assigning different residents for all weeks of the year. In other words, we solve the problem for a single week and the resulting schedule can be replicated for the remaining 51 weeks. There are various guidelines that need to be taken care while developing the clinic shift schedule. First, each resident can only be assigned to a single group. Second, there cannot be any empty groups. Third, a group can either be TWIG or OPD(TWIG and OPD are two types of clinics). Fourth, there should be at least one year 1 resident and one senior resident in each group. Fifth, each resident gets one day off amongst the 5 days the clinics are open. Finally, each resident can only do 1 full shift in a day.

Once the optimal groups are known with their respective resident assignments, they are used to solve the weekly schedule model to validate the results for the weekly resident rotation schedule.

Given the above, we solve the BIP model (explained in detail in Section 7.2) for finding: (a) What is the optimal number of clinic groups given the guidelines? (b) Which resident goes into which group (Here we assume that residents have no past history of clinic assignments)?

7.2 Binary Integer Programming Model (BIP Model)

Sets

$G1$ OPD Groups
$G2$ TWIG Groups
G All groups
S Shifts in a day
$R1$ Year 1 residents
RS Year 2 and year 3 residents
R R1 U RS
D Days in a week

Parameter

$h_{g,s}$ binary; 1 if group $g \in G$ is allowed to rotate in shift $s \in S$, and 0 o.w.
m maximum number of shifts in a week
n maximum number of shifts in a day
e_y minimum number of residents in a group from year y
q minimum number of residents in a group if open
a minimum number of OPD groups
b minimum number of TWIG groups

Decision Variables

$X_{r,g,d,s}$ binary; 1 if resident $r \in R$ from group $g \in G$ is assigned on day $d \in D$ to shift $s \in S$, 0 o.w.
Y_g binary; 1 if group $g \in G$ is open, 0 o.w.
$Z_{r,g}$ binary; 1 if resident $r \in R$ is assigned to group $g \in G$, 0 o.w.
$Q_{g,d,s}$ binary; 1 if group $g \in G$ rotates on day $d \in D$ in shift $s \in S$, 0 o.w.

Objective Function

Maximize 0

The objective for clinic group optimization problem is to obtain an optimal number of clinic groups. All the required guidelines were incorporated in the constraints and thus the objective function is kept as maximizing a constant.

Constraints

Constraint (1) ensures that each resident is assigned exactly one group.

$$\sum_{g \in G} Z_{r,g} = 1 \quad \forall \quad r \in R \tag{1}$$

Constraint (2) ensures that each resident is assigned to a group only when a group is open.

$$Y_g \geq Z_{r,g} \quad \forall \quad r \in R, \; g \in G \tag{2}$$

Constraint (3) ensures that if a group is open then at least one resident is assigned to that group.

$$\sum_{r \in R} Z_{r,g} \geq q \, Y_g \quad \forall \quad r \in R, \; g \in G \tag{3}$$

Constraint (4) ensures if a group is open then it is in rotation in every shift, provided it is allowed to rotate in that shift.

$$Q_{g,d,s} = Y_g \, h_{g,s} \quad \forall \quad d \in D, \; g \in G, \; s \in S \tag{4}$$

Constraint (5) ensures that at least one resident from year 1 is assigned to each open group

$$\sum_{r \in R1} X_{r,g,d,s} \geq e_y \, Q_{g,d,s} \, h_{g,s} \quad \forall \quad g \in G, \; s \in S, \; d \in D \tag{5}$$

Constraint (6) ensures that at least one resident from year 2 or year 3 is assigned to each open group

$$\sum_{r \in RS} X_{r,g,d,s} \geq e_y \, Q_{g,d,s} \, h_{g,s} \quad \forall \quad g \in G, \; s \in S, \; d \in D \tag{6}$$

Constraint (7) ensures that a resident can be assigned a shift only if the resident has been assigned a group

$$Z_{r,g} \, h_{g,s} \geq X_{r,g,d,s} \quad \forall \quad r \in R, \; g \in G, \; d \in D, \; s \in S \tag{7}$$

Constraint (8) ensures that each resident does m full clinic shifts in a week

$$\sum_{d \in D} \sum_{s \in S} X_{r,g,d,s} = m\ Z_{r,g}\ h_{g,s} \quad \forall \quad r \in R,\ g \in G \tag{8}$$

Constraint (9) ensures that each resident does exactly n full shifts each day

$$\sum_{s \in S} X_{r,g,d,s} \geq n\ h_{g,s}\ Y_g \quad \forall \quad r \in R,\ d \in D \tag{9}$$

Constraint (10) ensures that there is at least *a* open OPD groups

$$\sum_{g \in G1} Y_g \geq a \tag{10}$$

Constraint (10) ensures that there is at least *b* open TWIG groups

$$\sum_{g \in G2} Y_g \geq b \tag{11}$$

7.3 Experimentation and Results

The model described in the Section 4.3 is modeled on AMPL and solved on CPLEX. The solve time on an intel core i5 processor with 2 GB RAM is ~13 minutes. The optimal number of groups comes out to be 2: ***One TWIG and one OPD***.

The following table shows which resident allocations to the two groups :

Table 7.1 Optimal resident assignments to groups

Resident ID	**Assigned Clinic**
1,2,4,6,7,9,10,12,13,21,24,26,27,28,30,32,33,34,36,37,40,45,47,48,50,51,52,55	TWIG
3,5,8,11,14,15,16,17,18,19,20,22,23,25,29,31,35,38,39,41,42,43,44,46,49,53,54,56,57	OPD

8. EVALUATION OF OUTPATIENT CLINIC POLICIES

8.1 Methodology

This section is further divided into 3 sub sections. The first section talks about the outpatient clinic policies and the general methodological approach. The second section expands on how multiple problem instances are developed for experimentation. Finally the third section shows how Monte - Carlo simulation can be integrated into the methodology for obtaining more robust results.

8.1.1 Methodological approach

Clinic policies establish how frequently and for how long residents will rotate in their outpatient clinics to meet the minimum number of weeks in clinic rotations required by ACGME. Currently each resident is required to perform a week of clinic rotation after every 5 weeks (also known as the "4-1" policy"). From here onwards this policy will be referred as Policy A. This research explores the possibility of 2 other possible policy alternatives. The first policy, requiring residents to have 2 consecutive clinic rotations after 8 weeks will be referred as B. The second policy, allowing residents to have at least 1 clinic rotation every 5 weeks period (which is the minimum need as per ACGME); this will be referred as policy C. Thus, given these 3 policies, we explore which of the three policies result in the most balanced schedule.

The mathematical model considered for solving this problem is a special case of the weekly schedule model (WSM) used for solving the multi-objective optimization problem. The only change is that here we consider the weekly schedule model with only a single objective: maximize matching resident vacation preferences. This is done for simplification purposes as the focus of this part of the research is evaluation of outpatient clinic policies and hence we solve this part of the research as a single objective Mixed Integer Programming problem.

Solving different problem instances is part of the experimental set up. These instances are generated based on information provided by our partner hospital by randomizing resident's vacation preferences. Further detail regarding the process of generation of these instances is provided in the next sub section. Additionally, to compare different rotational schedules we assess each schedule over four metrics that account for key factors associated with having a balanced schedule. These factors are the number of night rotations, the number of elective rotations, the number of sick-call rotations, and how well the resulting schedule matches resident vacation preferences.

The following table consists of the summary of the metric:

Table 8.1: Metric for evaluation of schedules

Metric Factor	**What does it measure?**
Number of night shift rotation	Workload
Number of elective rotations	Resident education experience
Number of Sick-call rotations	Ability to provide patient care under unplanned resident absence
Overall vacation preference satisfaction	Resident satisfaction

8.1.2 Generating multiple problem instances

Different problem instances are generated by randomly generating different yet realistic resident vacation preferences. Residents in our partner hospital expressed their vacation preferences by assigning preference values that range from -3 to +9 to any given week of the academic year, which starts in the first week of June. Based on historical vacation preferences we simulate each resident's preference for any given week considering that his/her preference is uniformly distributed between the preference values shown in Table 8.2. These ranges for vacation preferences are a reflection of the information provided by our partner hospital.

Table 8.2: Vacation preference pattern across 52 weeks of the year

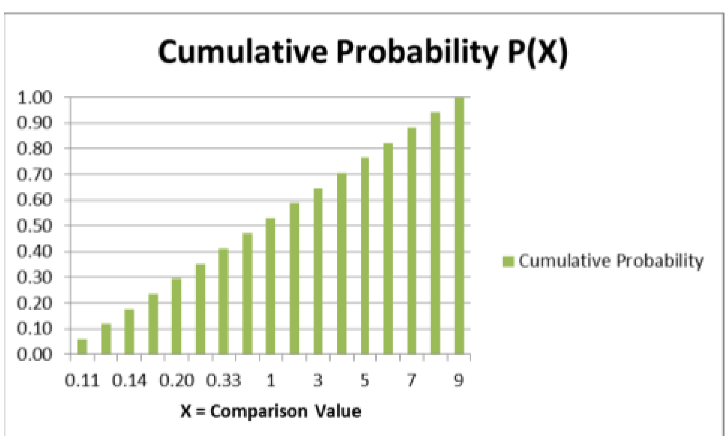
Weeks of the Resident Academic Year	Range of vacation preference values for the week
1-12 (June 1 – August 31)	-3 to 1
13-24 (September 1, November 30)	-1 to 3
25-34 (December 1 – February 7)	1 to 9
35-40 (February 8 – March 7)	-3 to 3
41-44 (March 8- April 7)	0 to 9
45-52 (April 8 – May 30)	-3 to 3

8.1.3 Monte Carlo Simulation Aided AHP

The results that will be obtained from the AHP described in the previous section will be based on the preferences of a single chief resident. Specifically speaking, the judgement matrix that compares the 4 criteria under consideration will take into consideration only a single chief resident's preference. Thus, in order to achieve more robust results we use a monte carlo simulation aided AHP approach. This involves the Monte-Carlo Simulation of the criteria pairwise comparison matrix. Although this does not represent multiple chief resident preferences, it does simulate multiple instances for a single chief resident. This approach provides a much more robust results in comparison to a single AHP.

The six cells highlighted (yellow) in figure 8.1 below can each take any of the values on Satty's scale, implying that we consider here an unbiased all possible comparisons of the 4 criteria. Since each of these cells can take any of the 17 values (1...9 and reciprocal of 1..9), we construct a discrete distribution that ensures random selection of values from the 17 specified values. The figure 8.2 below shows the discrete distribution that will be used for random selection of values.

Judgement Matrix 1	(1)	(2)	(3)	(4)
(1)	1	4	6	0.33333
(2)	0.25	1	8	0.33333
(3)	0.16666667	0.125	1	6
(4)	3	3	0.166667	1



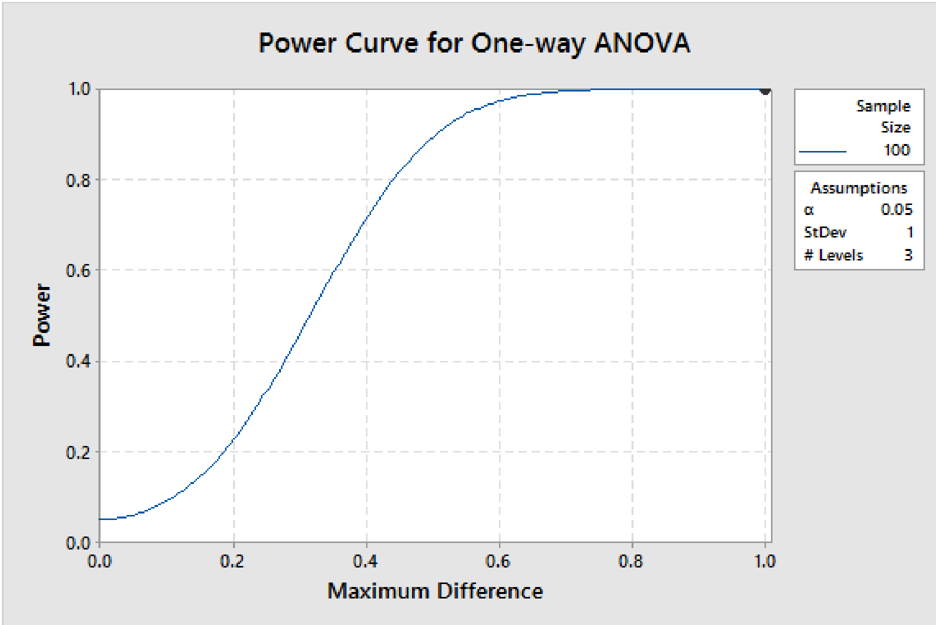
This distribution is then integrated with the AHP model with this random discrete distribution using VBA on MS Excel 2010. The model code is available in Appendix III. The resulting simulation tool is capable of automatic selection of values from the discrete distribution randomly, following which it carries out the rest of the AHP and gives out the ranking. Once the simulation tool is developed, we conduct multiple simulation runs for achieving robust results.

8.2 Experimentation and Results

This section describes the results used for analyzing various clinic scheduling policies and their effect on developing a balanced schedule. This section is divided in three sub sections. First section provides the experimentation details for conducting a single AHP assuming a single decision maker. Second provides the results from a single AHP. Third and the final sub-section provides the experimentation and results for monte-carlo simulation integrated AHP.

8.2.1 Experimentation

A special case of the WSM model is utilized for developing weekly rotation schedules for the outpatient clinic policy evaluation problem with the only change being in the objective function, where we treat this model as a single objective MIP problem, with the objective being maximization of vacation preferences. We test each policy for 100 problem instances each sharing same constraints but differing in the vacation preference expressed by the residents. Finally, we evaluate each 52-week schedule considering each of the 4 metrics described in Section 8.1.1. The number of scenarios tested for each policy ensures a sample size that will result in a power of test and type I error of 1.0 and 0.5, respectively which is reflected in the figure 8.3 below. Our experimentation was carried out by implementing the weekly schedule model (Section 4) on AMPL and solving it on Gurobi 5.6.0. on a Core i5 PC with 2 GB Ram**.**



Once we have the 100 schedules generated for each of the clinic policies, the three clinic policies are compared and ranked using the AHP. We first develop all pairwise comparison matrices based on the Satty's scale for the objectives, and then for each of the expected performance of the policies with respect to each objective. Once the weights for each objective and policy have been established, if consistent, they are fused into a single number representing the overall policy weight. These weights are used for policy ranking. The results and ranking are shown in the next sub-section.

8.2.2 Results from a single AHP

The follow section presents the results of the experimentation described through each of the pairwise comparison matrices (see Appendix II). The first judgement matrix is the pairwise comparison for the 4 decision criteria:

C_1 (Criterion 1): mean number night shift rotations per resident over 52 weeks, C_2 (Criterion 2): mean number elective rotations per resident over 52 weeks, C_3 (Criterion 3): mean number Sick-call rotations per resident over 52 weeks, C_4 (Criterion 4): Mean overall vacation preference satisfaction over 52 weeks.

Judgement Matrix for the 4 Criteria	**C_1**	**C_2**	**C_3**	**C_4**	**Priority Vector (W_{ij})**	**Rank**
C_1	1	5	5	2	0.526	1
C_2	0.200	1	1	0.500	0.112	3
C_3	0.200	1	1	0.333	0.101	4
C_4	0.500	2	3	1	0.262	2

$\lambda_{max} = 4.02$, C.I. = 0.005, C.R. = 0.57%

The above matrix is a result of a pairwise comparison of the 4 criteria based on the Satty's Scale. The nature of the comparisons for the purpose of this study is based on a single decision maker's perception (for future work we will consider multiple decision makers for obtaining a more robust matrix). The result shows that the most important criteria is C_1, followed by C_4, C_2, and finally C_3. Thus this implies that the most important criteria is the mean number night rotations per resident over 52 weeks and the least important criteria is the mean number Sick-call rotations per resident over 52 weeks. The comparisons made are consistent since the C.R. value is < 10%.

The following result is obtained on comparing the 3 policies on the mean number of night shift rotations done by each resident over 52 weeks.

C_1 (Criterion 1): Mean night shift rotations per resident over 52 weeks

Pairwise comparison Matrix for C_1	**Policy A**	**Policy B**	**Policy C**	**Priority Vector**	**Rank**
Policy A	1	0.995	1.024	0.335	2
Policy B	1.005	1	1.029	0.337	1
Policy C	0.977	0.972	1	0.328	3

$\lambda_{max} = 3$, C.I. = 0, C.R. = 0

The above results show that Policy B ranks slightly better than A, and policy A ranks better than C if only criteria 1 is used. This implies that when a resident schedule is assessed on number of mean night shift allocations per resident, policy B is the preferred clinic policy. As C.R. = 0, we know that the matrix is consistent in terms of the comparisons made.

The following result is obtained on comparing the 3 policies on the mean number of elective shift rotations done by each resident over 52 weeks.

C_2 (Criterion 2): Mean elective rotations per resident over 52 weeks

Judgement Matrix for C_2	**Policy A**	**Policy B**	**Policy C**	**Priority Vector**	**Rank**

Policy A	1	0.847	1.247	0.335	2
Policy B	1.181	1	1.473	0.396	1
Policy C	0.802	0.679	1	0.269	3

$\lambda_{max} = 3$, C.I. = 0, C.R. = 0

The results show that Policy B ranks better than A, and policy A ranks better than C. This implies that when a resident schedule is assessed on number of mean elective shift allocations per resident, policy B is the preferred clinic policy. As C.R. = 0, we know that the matrix is consistent in terms of the comparisons made.

The following result is obtained on comparing the 3 policies on the mean number of sick-call rotations done by each resident over 52 weeks.

C_3 (Criterion 3): Mean Sick-call rotations per resident over 52 weeks

Judgement Matrix for C_3	**Policy A**	**Policy B**	**Policy C**	**Priority Vector**	**Rank**
Policy A	1	1.932	0.723	0.345	2
Policy B	0.518	1	0.374	0.178	3
Policy C	1.383	2.671	1	0.477	1

$\lambda_{max} = 3$, C.I. = 0, C.R. = 0

The results show that Policy C ranks better than A, and policy A ranks better than B. This implies that when a resident schedule is assessed on number of mean Sick-call shift allocations per resident, policy C is the preferred clinic policy. As C.R. = 0, we know that the matrix is consistent in terms of the comparisons made.

The following result is obtained on comparing the 3 policies on the mean overall vacation preference satisfaction for residents over 52 weeks.

C_4 (Criterion 4): Mean vacation preference satisfaction value over 52 weeks

Judgement Matrix for C_4	**Policy A**	**Policy B**	**Policy C**	**Priority Vector**	**Rank**
Policy A	1	0.962	1.034	0.333	2
Policy B	1.040	1	1.075	0.346	1
Policy C	0.968	0.930	1	0.322	3

$\lambda_{max} = 3$, C.I. = 0, C.R. = 0

The results show that Policy B ranks better than A, and policy A ranks better than C. This implies that when a resident schedule is assessed on the mean vacation preference satisfaction value, policy B is the preferred clinic policy. As C.R. = 0, we know that the matrix is consistent in terms of the comparisons made.

The final matrix is a single priority weight matrix(A_{ij}) developed by appending the priority eigenvectors for each criterion into its columns:

$$\mathbf{A_{ij}} = \begin{bmatrix} 0.335 & 0.335 & 0.345 & 0.333 \\ 0.337 & 0.396 & 0.178 & 0.346 \\ 0.328 & 0.269 & 0.477 & 0.322 \end{bmatrix}$$

The above matrix(A_{ij}) is multiplied with the Priority eigenvector matrix(W_{ij}) obtained from the 'Judgement Matrix for the 4 Criteria:

$$\mathbf{W_{ij}} = \begin{bmatrix} 0.526 \\ 0.112 \\ 0.101 \\ 0.262 \end{bmatrix}$$

Therefore upon matrix multiplication : $\mathbf{(AW)_{ij}} = \sum_{k=1}^{4} A_{ik} W_{kj}$

Thus final priority vector, $\mathbf{(AW)_{ij}} = \begin{bmatrix} 0.336 \\ 0.330 \\ 0.335 \end{bmatrix}$

Upon normalizing to a scale on 100, the following final ranking is obtained:

Policy	**Final Priority Vector**	**Normalized Value**	**Final Rank**
A	0.336	100	1
B	0.330	98.31	3
C	0.335	99.69	2

The final result of the AHP is that Policy A is ranked as the better policy in comparison to Policy C, which in turn is better than Policy B.

The above results present the following findings. The following table summarizes the results:

Table 8.3: Ranking of clinic policies

Metric Measure	**Rank 1**	**Rank 2**	**Rank 3**
Workload	Policy B	Policy A	Policy C
Resident education quality	Policy B	Policy A	Policy C

Ability to provide patient care under unplanned resident absence	Policy A	Policy C	Policy B
Resident satisfaction	Policy B	Policy A	Policy C
Overall balance	Policy A	Policy C	Policy B

Thus when only a single chief resident preference is considered for pairwise comparison of the 4 criteria, policy A comes out as the best clinic policy that results in an overall balanced schedule.

8.2.3 Experimentation and results for Monte-Carlo Simulation Aided AHP

Once the decision support tool has been designed, it is found that not all possible set of random combinations of pairwise comparison values result in a C.R.(Consistency Ratio) of < 10%, which is a benchmark for checking consistency of the pairwise comparison matrix.

Thus we conduct a pilot study for analyzing how many simulation runs are acceptable(have a C.R. < 10%) from the ones that are done. The following table summarizes the results:

Table 8.4: Pilot Study

Total Simulation Runs	Runs with C.R. < 10%
20	2
40	1
100	2
500	5
1000	17
2000	30

Thus given the nature of the simulation and the time constraint, a total of **64,713 simulations** were conducted. Out of these **1150 runs** were deemed acceptable. It was on these runs that the analysis was done. For initial analysis one way ANOVA (Analysis of Variance) was used with the following conditions:

Null hypothesis: Means of policies are equal

Significance level: $\alpha = 0.1$

Equal variances were not assumed for this analysis.

The following table shows the results that were obtained:

Table 8.5: ANOVA results

Policy	# of replicates	Mean	Std. deviation
A	1150	93.81	5.27
B	1150	88.05	13.76
C	1150	96.63	5.16

The above results show Policy C as the best policy but it is seen that the data for this policy does not follow the normality assumption implying that ANOVA is not valid for this experiment. Since the data set for this experiment is found to be of non-parametric nature, we conduct a Kruskal -Wallis Test. The following table summarizes the results of the Kruskal-Wallis Test

Table 8.6: Kruskal-Wallis Test results

Policy	# of replicates	Median
A	1150	95.10
B	1150	95.23
C	**1150**	**100.00**

The final result of the simulation aided AHP is that policy C comes out as the best policy. This experimentation shows the importance of the criteria pairwise comparison matrix and the uncertainty associated with it. Although we do not claim to have considered all possible preferences, we can conclude that for the scope of this study the "at least one clinic in any 5 weeks" results in the most balanced schedule when multiple preferences of a single chief resident is considered

9. CONCLUSION

The multi-objective optimization approach for developing year long resident rotation provides schedules that not only follow the guidelines imposed by the official governing bodies and the hospital, but are also are well balanced in terms of workload, education experience, satisfaction and ability to provide patient care under unplanned residence absence. This approach coupled with the AHP provides a system that is capable of generating multiple feasible schedules that can be evaluated for balance based on the decision maker's priorities. The resulting schedules are much more robust than traditional approaches because the data for the AHP is taken from experimentation rather than from subjective decisions, thereby making our proposed methodology a very robust approach. Also, this approach is very much scalable and adaptable to other healthcare institutions who might be interested in developing schedules that are balanced with their priorities.

The integrated approach of reducing infeasibilities in weekly and daily schedules provides insight into the fact that identification of certain key restrictive units for both weekly and daily schedules can be used for development of better and more feasible weekly rotation schedules, which henceforth can result in feasible daily schedules. This approach of eliminating infeasibility issues is better than the conventional approach of to and fro solving of weekly and daily schedules, specially in terms of the solve times. The approach adds hard constraints to the weekly schedule model, hence the increase in solve times. The decrease in variance in the night shifts is justified since we have a majority of all the night shift units included in the set of restrictive units for solving the weekly schedule using the integrated approach. Overall, this integrated approach of using data analysis and optimization in a combination presents an effective way of reducing infeasibilities in weekly as well as daily schedules.

Finding the optimal number of clinic groups problem results in a system that has an enhanced focus on continuity of care because with the new optimal assignment of groups, there is never a week when there are no a resident doing a clinic rotation from a particular group. The results obtained from the BIP model (section 6) are used to resolve the single objective MIP model (section 4) and it is observed that continuity of care is improved from previous schedules with the 2 groups, which effectively means that there is always some resident from each group in each week, thereby ensuring that all patients are taken care of irrespective of the fact whether their respective resident is in rotation or not. Although it should be mentioned here that the BIP model makes 2 major assumption. The first assumption is that proctors or senior doctors who supervise are available at all times and can supervise any number of residents. The second assumption is that residents work one full shift in a day. Thus any change in these two assumptions can possibly modify the optimal number of groups as well the assignments of residents to those groups.

Finally, the outpatient clinic policy evaluation result for a single AHP analysis show that policy C is the top choice for developing a balanced schedule, although the difference in the final weight values for all the three policies is insignificant, implying that essentially no particular policy can be favored from that analysis. But since this result is based on a single preference for the pairwise judgement comparison matrix, the requirement of simulation aided AHP is imminent, which ultimately provides a more thorough and robust ranking of the clinic policies. The clinic policy ranking obtained from the monte-carlo simulation aided AHP considers multiple instances of a single chief resident's preference. This aspect of the research in itself can be seen as an interesting conclusion due to the limitation of considering data from all hospitals. Since each hospital has its own set of specific requirements and preferences, it virtually becomes impossible to incorporate all possible constraints and preferences from all hospitals to provide the most robust solution possible, majorly because there probably doesn't exist an overall solution that fits the needs of all hospitals while agreeing with the preferences of all chief residents. Development of resident rotation schedules using Policy C can have tremendous impacts on resident scheduling process because since the clinic constraints fall into the category of hard constraints, and since policy C is the most relaxed clinic policy, schedules developed using Policy C can result in resident rotation schedules which are more flexible in terms of ambulatory rotations for the residents and at the same time maintain continuity of care. Thus a schedule generated keeping policy C in consideration will be less restrictive than those generated by the other policies since it just ensures the minimum ACGME guideline for clinic rotations, which is at least 1 clinic in 5 weeks. However, the lack of a clear pattern clinic visits for Policy C schedules, may affect the clinics ability to organize instructional events for its residents.

Overall, this research presents some novel optimization based approaches for developing year long weekly resident rotation schedules, daily rotation schedules as well as clinic shift schedules. What makes this research unique from other previously tried approaches is that it combines the optimization aspect of the research with various other elements like simulation, AHP and heuristics that overall helps to provide a very robust system for not only development of balanced resident rotation schedules but also their evaluation on different multi-criteria assessments which can be based on the decision makers preferences. In a nutshell this research provides a comprehensive overall mechanism for developing balanced resident rotation schedules that keeps the administration, the care givers as well as the patients happy.

10. FUTURE WORK

Finding the optimal number of residents for the residency program is viable extension of this research. The optimal number residents would help further reduce the infeasibilities in the schedule and could possibly result in development of a schedule that is closest to an optimal resident rotation schedule.

Solving the outpatient clinic policies evaluation problem as a multi-objective optimization problem is an interesting piece of future work as the goal programming approach can provide much more robust rankings for the policies.

Another extension of this research can be in the direction of considering requests of residents for not just vacations but also for all other units where they might be assigned to rotate. This approach will add another dimension to resident satisfaction as this approach would be more focused on maximizing resident requests for vacations as well as other their requests for rotating in specific units in their preferred weeks.

Another potential future work would be a multi-layered AHP integrated with monte-carlo simulation for evaluation of outpatient clinic policies. Currently, we use monte-carlo simulation to simulate multiple preferences of a single chief resident but in case of multiple chief residents, we would need to have a multi-layered AHP where there would be an AHP for ranking the chief residents which would then be integrated with our current AHP.

Finally, one final crucial area of future research can be modification of the objective function in the Weekly Schedule model(WSM) for developing even more robust and balanced weekly resident rotation schedules. This change would involve minimization of variance in night shift rotation, elective rotations and so on. Having said that, the challenge here will be to avoid the development of a non linear mixed integer programming problem.

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APPENDECIES

Appendix I. Set of all units

Γ : {TWIG, OPD, MICU_D, MICU_N, Electives, Seetharaman Floor, Heme Floor, Cards Floor, Float Floor, TBC1, TBC2, MAT_D, MAT_N, Midcall, Overnight, RNAT}

Appendix II. Analytical Hierarchy Process

AHP is a methodology first designed and proposed by Saaty [21], which aims to compare how multiple alternatives are compared with each other over multiple criteria, and when the criteria are not necessarily commensurable. In our case alternatives correspond to candidates which we want to ultimately rank and the criteria to the objectives on which we want to evaluate these alternatives. As an example, for answering the research question related to evaluation of clinic policies, the alternatives correspond the various clinic policies while the criteria would be the factors in the metric described in section 3.4

AHP relies on defining pair-wise comparison matrices. First for comparing how a decision maker assess the importance of each criteria against each other, and then on how each alternative performs against other alternatives for a given criterion. The use of these matrices permits determining the weights for each alternative regarding under the optic of each criterion, and also establishing the weights of the criterion. Fussing these weights allows determining the actual importance of each alternative. A pairwise comparison matrix is represented by

$$A=[a_{ij}] \qquad where \qquad a_{ij}=\frac{1}{a_{ji}} \tag{1}$$

In order to perform the pair-wise comparisons, Satty suggests the scale listed in Table 1. The comparison

a_{ij} corresponds to an estimation of the ratio between $\frac{w_i}{w_j}$ (i.e., the actual importance (or weight) of i over the importance of j)

It is shown below:

Level of Importance	**Meaning**
1	The two activities are of equal importance
3	One activity is moderately more important than the other
5	There is a strong importance of one activity over the other
7	One activity has a very strong importance in comparison with the other
9	One activity is extremely important than the other
2,4,6,8	Intermediate importance between two adjacent decisions
Reciprocals of 1..9	If for any pairwise comparison activity j is more importance to activity i

Table 1: Scale of relative importance

Pair-wise comparison matrices are symmetrical so an entry $a_{ij}=\frac{1}{a_{ji}}$ (2)

Consequently the A pairwise comparison matrix is defined by:

$$A=[a_{ij}]=\left[\frac{w_i}{w_j}\right] \quad \textit{where} \quad w_i \textit{ is the absolute importance of } i \ (\textit{which is unknown}). \tag{3}$$

Hence,

$$A\,W=\lambda\,W \quad , \tag{4}$$

where W is the vector of importance of the alternatives (or criteria) under comparison. Then, W correspond to the eigenvector of the system of equations.

The right principal eigenvector (also called as the priority vector) for each of these pair-wise comparison matrices can be approximated by the geometric mean of each row in the matrix, or by dividing each element of the matrix by the sum of its column, following which the average across the rows is calculated. In this study, we use the later approach in this study and the following equations show the same:

Hence, for the pairwise comparison matrix A, , the eigenvector W is given by

$$W=\left[\frac{1}{n}\sum_{j=1}^{n}\frac{a_{ij}}{\sum_{i}a_{ij}}\right] \qquad where \qquad n\ is\ the\ number\ of\ columns\ \text{in matrix}\ A \tag{5}$$

In addition to calculating the eigenvectors for each matrix, we must also find how consistent are the pairwise comparisons for each comparison matrix.

Consistency Ratio or CR is measure of the logical consistency of the pair-wise comparisons. CRs above 10% indicate that the pairwise comparisons need to be revised. The following equations describe the procedure for calculating the CR for a Judgement matrix:

First we need to find λ_{max} :

Consider the matrix in equation 1 and eigen vector in equation 4, then $\lambda_{max}=\left[\sum_{j}a_{ij}\right]\left[W_{ij}\right]$ (6)

The next step is to calculate Consistency Index or CI = $\dfrac{(\lambda_{max}-n)}{(n-1)}$ (7)

Finally CR is calculated by dividing CI by Random Consistency Index or RCI. The following table is used as a standard for calculating RCI values depending on the value of n:

n	1	2	3	4	5	6	7	8	9

RCI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Once it is established that the pair-wise matrices are consistent and the eigenvectors have been calculated, the final step is executed. This step involves making a single decision matrix consisting of all the criteria eigenvectors appended to its columns. Then this matrix is multiplied with the eigenvector obtained from the Judgement matrix that compared all the criteria. The resulting matrix is the final priority matrix or the final eigenvector, that can be used to rank the alternatives based on the priority vector values. The following equation represents the final priority values for m alternatives and n criteria:

$$W^{i}_{AHP} = \sum_{j \in 1}^{n} W_i^j W_j^o \qquad for \qquad i = 1,2,3\ldots m \tag{8}$$

Here W_j^o is the eigen vector for the pairwise comparison of the n criteria while W_i^j is the eigen vector for the pairwise comparison of each alternative on each criteria.

Appendix III. Simulation Integrated AHP Visual Basic Code

The code is written in MS Excel Visual Basic(VBA). The basic working of the code is that it asks for the number of simulations required(in the GUI created in the MS Excel Model) following which it loops for that number. In each loop it populates the yellow cells in Figure 2(in Section 3) with values taken from a random discrete distribution(Figure 3 in Section 3). Following which it records the required data in another sheet.

```
***Start Program***
Sub Experimental_Runs()
Dim numruns As Variant
numruns = InputBox("Enter number of runs")
For i = 1 To numruns
Sheet1.Activate
Range(Cells(44, 4), Cells(44, 9)).Copy
Range(Cells(45, 4), Cells(45, 9)).PasteSpecial Paste:=xlValues
```

```
Cells(45, 4).Copy
Cells(2, 3).PasteSpecial Paste:=xlValues
Cells(45, 5).Copy
Cells(2, 4).PasteSpecial Paste:=xlValues
Cells(45, 6).Copy
Cells(2, 5).PasteSpecial Paste:=xlValues
Cells(45, 7).Copy
Cells(3, 4).PasteSpecial Paste:=xlValues
Cells(45, 8).Copy
Cells(3, 5).PasteSpecial Paste:=xlValues
Cells(45, 9).Copy
Cells(4, 5).PasteSpecial Paste:=xlValues
Range(Cells(45, 4), Cells(45, 9)).Copy
Sheet2.Activate
If IsEmpty(Cells(2, 1)) Then nextemptycell = 2 Else nextemptycell = Cells(1, 1).End(xlDown).Row + 1
Cells(nextemptycell, 1).PasteSpecial Paste:=xlValues
Sheet1.Activate
Range(Cells(52, 4), Cells(52, 6)).Copy
Sheet2.Activate
Cells(nextemptycell, 7).PasteSpecial Paste:=xlValues
Sheet1.Activate
Range("Z14").Copy
Sheet2.Activate
Cells(nextemptycell, 10).PasteSpecial Paste:=xlValues
Next i
End Sub
***End Program***
```

Appendix IV. Weekly Schedule Model with Artificial Variables

Objective Function

The following objective function is the full objective function with the sum of penalties for all soft constraints. To avoid repetition only maximizing matching vacation preferences has been shown along with the penalties. Similar objective functions can be visualized in case of a multi-objective problem.

$$\text{Maximize} \sum_{r\in\chi}\sum_{u\in V}\sum_{t\in T} p_{14}\Psi_{r,t}X_{r,u,t} \; - \; p_i\sum_{r\in\chi}\sum_{i=1}^{3}\gamma_r^i \; - \; p_i\sum_{u\in N}\sum_{t\in T}\sum_{i=4}^{5}\gamma_{u,t}^i$$

$$-\,p_i\sum_{u\in U}\sum_{t\in T}\sum_{i=6}^{7}\gamma_{u,t}^i \; - \; p_i\sum_{j\in K:\,j\in H_u}\sum_{r\in R_j}\sum_{u\in U}\sum_{i=8}^{11}\gamma_{j,r,u}^i \; - \; p_{12}\sum_{j\in K:\,j\in H_u}\sum_{r\in R_j}\sum_{u\in U}\sum_{t\in T}\gamma_{j,r,u,t}^{12}$$

$$-\,p_{13}\sum_{r\in\chi}\sum_{u\in V}\sum_{t\in 1..np-2}\gamma_{r,u,t}^{13} \; - \; p_{15}\sum_{r\in\chi}\sum_{t\in T}W_{r,\text{'SickCall'},t}$$

Constraints

Hard Constraints

$$W_{r,u,t} \;\geq\; W_{r,u,t}^o \qquad \forall\; r\in\chi,\; u\in U,\; t\in T \tag{1}$$

$$X_{r,u,t} \;\geq\; X_{r,u,t}^o \qquad \forall\; r\in\chi,\; u\in U,\; t\in T \tag{2}$$

$$\Delta_{u,t} \;\geq\; \Delta_{u,t}^o \qquad \forall\; u\in U,\; t\in T \tag{3}$$

$$\sum_{n\in\alpha_c^{min}} X_{(r,c,t+\pi_c n)} \;\geq\; W_{r,c,t} \qquad \forall\; r\in\chi,\; t\in 1..\Pi,\; c\in C \tag{4}$$

$$\sum_{t}^{t+\pi_c-\gamma_c} X_{r,c,t} \;=\; \gamma_c \qquad \forall\; r\in\chi,\; t\in 1..np-\pi_c-\gamma_c,\; c\in C,\; g\in G \tag{5}$$

$$\sum_{c\in C}\sum_{t=1}^{t=\Pi} W_{r,c,t} \;=\; s \qquad \forall\; r\in\chi \tag{6}$$

$$\sum_{g\in G} l_{r,g}h_{g,c} - X_{r,c,t} \;\geq\; 0 \qquad \forall\; r\in\chi,\; t\in T,\; c\in C \tag{7}$$

$$\sum_{r\in\chi}\sum_{c\in C} l_{r,g}W_{r,c,t} \;\geq\; m \qquad \forall\; t\in 1..\Pi,\; g\in G \tag{8}$$

$$\sum_{r\in\chi:\,l_{r,g}=1} X_{r,c,t} \;\geq\; \Omega_{g,t} \qquad \forall\; g\in G,\; c\in C,\; t\in T : h_{g,c}=1 \tag{9}$$

$$\sum_{r\in R_i} X_{r,u,t} \;\leq\; \Phi_{i,u}^{max} \qquad \forall\; u\in U,\; t\in T,\; i\in 1..3 \tag{10}$$

$$\sum_{u\in U} X_{r,u,t} \;=\; w \qquad \forall\; r\in\chi,\; t\in T \tag{11}$$

$$\sum_{r\in R_j} X_{r,u,t} = 0 \qquad \forall\ j\in \mathrm{K},\ t\in T,\ u\in U : j\notin \mathrm{H}_u \tag{12}$$

$$X_{r,u,t} - \sum_{tt\in(t-\lambda_u^{min}+1)..t} W_{r,u,tt} = 0 \qquad \forall\ j\in \mathrm{K},\ r\in\chi,\ u\in U,\ t\in\lambda_u^{min}..np : j\in \mathrm{H}_u \tag{13}$$

$$\sum_{u\in UN}\sum_{t_o\in t..t+3} X_{r,u,t_o} \le l \qquad \forall\ r\in\chi,\ t\in 1..np-3 \tag{14}$$

Soft Constraints

$$\sum_{t\in T}\sum_{u\in A} X_{r,u,t} + \sum_{u\in A}\omega_{r,u} + \gamma_r^1 \ge (3np-12)/3 \qquad \forall\ r\in\chi \tag{15}$$

$$\sum_{t\in T}\sum_{u\in E} X_{r,u,t} + \sum_{u\in E}\omega_{r,u} + \gamma_r^2 \ge (3np-12)/3 \qquad \forall\ r\in\chi \tag{16}$$

$$\sum_{t\in T}\sum_{u\in I} X_{r,u,t} + \sum_{u\in I}\omega_{r,u} + \gamma_r^3 \ge (3np-12)/3 \qquad \forall\ r\in\chi \tag{17}$$

$$\sum_{j\in \mathrm{H}_u}\sum_{r\in R_j :\ j\le 3} X_{r,u,t} + \gamma_{u,t}^4 \ge k \qquad \forall\ u\in N,\ t\in T \tag{18}$$

$$\alpha_u^{min} \le \sum_{t\in 1..np} X_{r,u,t} + \gamma_{j,r,u}^8 \qquad \forall\ j\in \mathrm{K},\ r\in R_j,\ u\in U : j\in \mathrm{H}_u \tag{19.1}$$

$$\alpha_u^{max} \ge \sum_{t\in 1..np} X_{r,u,t} + \gamma_{j,r,u}^9 \qquad \forall\ j\in \mathrm{K},\ r\in R_j,\ u\in U : j\in \mathrm{H}_u \tag{19.2}$$

$$\sum_{j\in \mathrm{H}_u}\sum_{r\in R_i} X_{r,u,t} + \gamma_{u,t}^j \ge \Phi_{i,u}^{min} \qquad \forall\ u\in Q,\ t\in T,\ i\in 1..3,\ j\in 5..7 \tag{20}$$

$$\zeta_u^{min} \leq \sum_{t \in T} X_{r,u,t} + \omega_{r,u} + \gamma_{j,r,u}^{10} \quad \forall \ j \in \mathrm{K}, \ r \in R_j, \ u \in U : j \in \mathrm{H}_u \tag{21.1}$$

$$\zeta_u^{max} \geq \sum_{t \in T} X_{r,u,t} + \omega_{r,u} + \gamma_{j,r,u}^{11} \quad \forall \ j \in \mathrm{K}, \ r \in R_j, \ u \in U : j \in \mathrm{H}_u \tag{21.2}$$

$$\sum_{(tt \in t..t+\lambda_{u-1}^{min} : \ t+\lambda_{u-1}^{min} \leq np)} X_{r,u,tt} + \gamma_{j,r,u,t}^{12} \geq W_{r,u,t} \lambda_u^{min} \quad \forall \ j \in \mathrm{K}, \ r \in R_j, \ u \in U, \ t \in T : j \in \%ET_u \wedge \lambda_u^{min} > 1 \tag{22}$$

$$W_{r,u,t} + W_{r,u,t+2} + \gamma_{r,u,t}^{13} \leq 1 \quad \forall \ r \in \chi, \ u \in V, \ t \in 1..np-2 \tag{23}$$

Appendix V. RGH Data

Table 1. Resident Info

Resident Year	Resident ID
1	1
1	2
1	3
1	4
1	5
1	6
1	7
1	8
1	9
1	10
1	11
1	12
1	13
1	14
1	15
1	16
1	17
1	18
1	19

2	20
2	21
2	22
2	23
2	24
2	25
2	26
2	27
2	28
2	29
2	30
2	31
2	32
2	33
2	34
2	35
2	36
2	37
2	38
3	39
3	40
3	41
3	42
3	43
3	44
3	45
3	46
3	47
3	48
3	49
3	50
3	51
3	52
3	53
3	54
3	55
3	56
3	57

Table 2. Data for past rotations in units by senior residents

Resident ID	CardsFloor	Electives	Geriatrics	HemeFloor	MICU_D	MICU_N	OPD	SeetharamanFloor	TBC1	TBC2	FloatFloor	TWIG	VAC
1	2	10	0	4	4	2	9	4	2	2	2	0	4
2	2	9	0	4	4	2	10	2	4	4	2	0	4
3	2	11	0	4	4	2	9	2	1	3	2	0	4
4	4	9	0	2	4	4	9	2	2	0	5	0	4
5	2	9	0	2	4	2	10	4	2	3	4	0	4
6	2	9	0	6	6	2	9	2	2	4	2	0	4
7	2	9	0	2	4	2	9	4	2	6	2	0	4
8	2	8	0	4	6	2	9	2	2	2	4	0	4
9	6	9	0	2	4	4	8	2	5	0	2	0	4
10	2	8	0	2	6	2	9	2	5	2	3	0	4
11	2	9	0	2	6	2	9	4	2	4	2	0	4
12	4	7	0	2	6	2	0	3	2	4	0	10	4
13	5	7	0	2	5	2	9	2	5	2	0	0	4
14	2	9	0	2	4	2	0	4	0	4	4	9	4
15	4	9	0	2	6	2	0	1	2	4	2	9	4
16	4	7	0	1	4	4	0	2	2	4	2	10	4
17	4	8	0	2	6	2	0	2	3	2	2	9	4
18	2	8	0	3	4	4	0	2	2	4	2	9	4
19	1	10	0	4	5	2	0	4	3	0	4	9	4
20	**CardsFloor**	**Electives**	**Geriatrics**	**HemeFloor**	**MICU_D**	**MICU_N**	**OPD**	**SeetharamanFloor**	**TBC1**	**TBC2**	**FloatFloor**	**TWIG**	**VAC**
21	2	23	4	8	8	10	20	6	6	0	3	0	8
22	2	28	3	6	6	12	0	6	6	0	2	21	8
23	4	31	3	6	10	6	0	6	2	0	2	20	8
24	4	24	4	4	10	6	21	6	3	0	4	0	8
25	3	24	4	4	8	10	0	4	8	0	2	20	8
26	4	29	4	2	6	10	0	7	2	0	4	21	8
27	4	26	3	4	10	8	0	6	3	0	4	20	8
28	2	25	4	8	10	10	20	5	6	0	2	0	8
29	4	26	4	8	8	10	21	2	2	0	2	0	8
30	2	26	4	2	6	8	21	6	8	2	2	0	8
31	4	28	2	4	8	8	0	6	4	0	4	20	8
32	2	26	4	4	8	6	20	8	8	0	2	0	8
33	0	32	1	4	6	8	0	4	8	0	0	21	8
34	2	31	2	4	6	10	0	10	2	0	4	21	8
35	4	28	4	6	10	6	0	4	4	0	2	20	8
36	4	24	4	6	10	10	21	2	6	0	2	0	8

37	4	27	3	3	8	8	20	7	6	0	2	0	8
38	1	29	3	6	8	8	0	4	8	2	2	20	8
39	2	26	3	6	8	8	19	6	6	0	2	0	8

Table 3. Unit Types

Ambulatory	**Inpatient**	**Elective**
TWIG	MICU_D	Electives
OPD	MICU_N	Research
Geriatrics	SeetharamanFloor	
	HemeFloor	
	CardsFloor	
	FloatFloor	
	TBC1	
	TBC2	
	MAT_D	
	MAT_N	
	Midcall	
	Overnight	
	RNAT	

Table 4. Clinic Groups

Clinic Group	**Clinic Type**
Light Blue	TWIG
Purple	OPD
Green	TWIG
Yellow	OPD
Orange	TWIG

Table 5. Resident pre assignment to clinic groups

Resident ID	**Clinic Group**
1	Yellow
2	Blue
3	Yellow
4	Orange
5	Orange

6	Orange
7	Blue
8	Green
9	Blue
10	Blue
11	Green
12	Yellow
13	Yellow
14	Blue
15	Blue
16	Green
17	Green
18	Green
19	Orange
20	Yellow
21	Yellow
22	Yellow
23	Yellow
24	Yellow
25	Yellow
26	Yellow
27	Purple
28	Purple
29	Purple
30	Purple
31	Purple
32	Purple
33	Light Blue
34	Light Blue
35	Light Blue
36	Orange
37	Orange
38	Orange
39	Orange
40	Light Blue
41	Light Blue
42	Light Blue
43	Purple
44	Purple
45	Purple
46	Yellow

47	Yellow
48	Yellow
49	Yellow
50	Yellow
51	Yellow
52	Green
53	Green
54	Green
55	Green
56	Orange
57	Orange

Table 6. Resident requirements in units

CardsFloor	**Minimum R1**	**Minimum R2**	**Minimum R3**	**Max R1**	**Max R2**	**Max R3**
TWIG	1	1	1	3	3	3
OPD	1	1	1	3	3	3
MICU_D	2	1	1	2	1	1
MICU_N	1	1	1	1	1	1
Geriatrics	0	1	0	0	2	0
SeetharamanFloor	1	0	1	2	0	1
HemeFloor	1	1	0	1	1	0
CardsFloor	1	0	1	2	0	1
FloatFloor	1	1	0	1	1	0
TBC1	1	1	0	1	1	0
TBC2	1	0	1	1	0	1
MAT_D	0	1	1	0	1	1
MAT_N	0	1	1	0	1	1
Electives	3	2	2	19	19	19
RNAT	0	0	1	0	0	1
Overnight	1	1	0	1	1	0
Midcall	0	1	0	0	1	0
Minimum and Maximum residents required each week in clinic groups						
Light Blue	1	1				
Purple	1	1				
Green	1	1				
Yellow	1	1			4	
Orange	1	1				

Table 7. Rotations required in units

Unit	Minimum Continuous Rotation	Max Continous Rotation	Minimum rotations in 1 year	Maximum rotations in 1 year	Minimum rotations in 3 years	Maximum rotations in 3 years
TWIG	1	1	10	11	30	33
OPD	1	1	10	11	30	33
MICU_D	2	2	2	8	12	24
MICU_N	2	2	2	8	12	24
Vac	2	2	4	4	12	12
Geriatrics	1	4	1	4	1	4
SeehtaramanFloor	2	4	2	4	6	12
HemeFloor	2	4	2	4	6	12
CardsFloor	2	4	2	4	6	12
FloatFloor	2	4	2	4	6	12
TBC1	2	4	2	4	6	12
TBC2	2	4	2	4	6	12
MAT_D	2	2	2	.	4	.
MAT_N	2	2	2	.	4	.
Electives	2	4	2	.	6	.
RNAT	2	4	4	4	0	.
Overnight	2	2	4	4	0	.
Midcall	2	2	4	4	0	.

Table 8. Units requiring residents in rotation each week

Units requiring residents in rotation each week
MICU_D
MICU_N
SeetharamanFloor
HemeFloor
CardsFloor
FloatFloor
TBC1
TBC2
MAT_D
MAT_N
RNAT

Overnight
Midcall

Table 9. Resident type and their allowed rotations

Resident Year	TWIG	OPD	MICU_D	MICU_N	Geriatrics	SeetharamanFloor	HemeFloor	CardsFloor	FloatFloor	TBC1	TBC2	MAT_D	MAT_N	Electives	RNAT	Overnight	Midcall
1	Y	Y	Y	Y		Y	Y	Y	Y	Y	Y			Y		Y	
2	Y	Y	Y	Y	Y		Y		Y	Y		Y	Y	Y		Y	Y
3	Y	Y	Y	Y		Y		Y			Y	Y	Y	Y	Y		